MoM/BI–RME Analysis of Boxed Microwave Circuits Based on Arbitrarily Shaped Elements

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Abstract— **In this paper, we propose a novel approach for the analysis of shielded microstrip circuits, composed of a number of thin metallic areas with arbitrary shapes and finite conductivity, embedded in a multilayered lossy medium. The analysis is based on the solution of an Integral Equation (IE) obtained by enforcing the proper boundary condition to the electric field on the metallic areas. The solution of the IE is obtained by the Method of Moments (MoM) with entire domain basis functions, which are numerically determined by the Boundary Integral–Resonant Mode Expansion (BI–RME) method. The use of the BI–RME method allows for the efficient calculation of entire domain basis functions in the case of metallic areas with arbitrary shapes, thus permitting the analysis of a wide class of circuits. Two examples demonstrate the accuracy, rapidity, and flexibility of the proposed method.**

I. Introduction

In the last years, a considerable interest has been directed to the design of boxed multilayered circuits. This configuration is typically found in many actual Monolithic Microwave Integrated Circuit (MMIC) designs, which place the printed elements in several layers to achieve very compact designs [1].

An efficient approach for their analysis is based on the Integral Equation (IE) method, usually coupled to the Method of Moments (MoM) with subdomain basis functions (typically roof–tops) [2]. While the use of subdomain basis functions is very general in application without any restriction in the shape of the metallic areas, it leads to the solution of large matrix problems.

Recently, the IE/MoM method was applied with entire domain basis functions, in the case of shielded microstrip circuits, consisting of rectangular metallic areas in a multilayered dielectric medium [3]. This method is very efficient, since few entire basis functions are needed, but it is limited to areas with a canonical shape (*e.g.*, rectangular, circular), where the entire domain basis functions are known analytically.

In this paper, we present the extension of the method proposed in [3] to the case of metallic areas with arbitrary shape (Fig. 1). The entire domain basis functions are determined numerically by the Boundary Integral–Resonant Mode Expansion (BI–RME) method [4]. The idea of using the BI–RME method has two main advantages. The first is the possibility of using entire domain basis functions instead of roof-tops for arbitrary shapes, and the second is that the unknown of the whole problem are the currents on the boundary of the metallic areas instead of the complete surface. This allows the reduction of the size of the matrices, speeds–up their calculation, and permits the design of substantially more complex circuits.

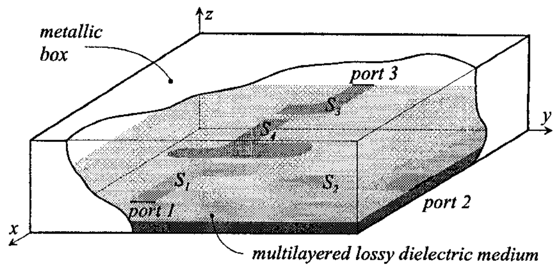

Fig. 1. Example of a shielded microstrip circuits, with arbitrarily shaped metallic areas in a multilayered lossy medium.

II. IE/MoM Approach

Let us consider the structure depicted in Fig. 1, consisting of a multilayered medium and P metallic areas with arbitrary shapes S_p. The circuit is fed at the frequency ω by delta–gap excitations at N ports, defined on areas contacting the walls of the shielding box.

In [3], the analysis is based on the solution of a system of P integral equations, which are obtained by enforcing the boundary condition for the transverse electric field at all the metallic areas S_p $(p = 1, \dots, P)$

$$Z_s \vec{J}_p(\vec{r}) - \sum_{q=1}^{P} \int_{S_q} \overleftrightarrow{G}(\vec{r}, \vec{r}\,'|\omega) \cdot \vec{J}_q(\vec{r}\,')\, dS' = \vec{E}_p(\vec{r}) \quad (1)$$

where: $\vec{r}$ belongs to S_p; Z_s is the surface impedance of the conductors; $\vec{J}_p$ is the (unknown) current density on S_p; $\vec{E}_p$ is the exciting electric field applied to the port on the p–th patch ($\vec{E}_p = 0$ for internal patches); $\overleftrightarrow{G}$ is the multilayered medium Green's function, formulated in the spectral domain using the well known modal series expansion in the cavity enclosing the circuit

$$\overleftrightarrow{G}(\vec{r}, \vec{r}'|\omega) = \sum_m V_m(z, z'|\omega)\, \vec{\mathcal{E}}_m(x, y)\, \vec{\mathcal{E}}_m(x', y') \quad (2)$$

where V_m are voltages computed in equivalent transverse transmission lines networks representing the layered medium as described in [5], and $\vec{\mathcal{E}}_m$ are the transverse electric modal vectors of the shielding enclosure.

The integral equations (1) are solved with the MoM, which leads to a linear matrix problem. A key feature of the proposed approach is to choose as basis and test functions $\vec{e}_r^{(p)}$, which span the entire domain S_p (see Fig. 1). Since the current $\vec{J}_p$ is tangential to the boundary but to the port segments, and perpendicular to the port segments, for each area S_p a suited set of entire domain basis functions $\vec{e}_r^{(p)}$ are the electric modal vectors of a waveguide with a cross–section S_p, bounded by magnetic and, possibly, electric walls [3]. In particular, for internal patches (*e.g.*, S_4 in Fig. 1), the boundary condition is a perfect magnetic wall on the whole boundary ∂S_p. Conversely, for patches attached to the box wall (*e.g.*, S_1, S_2, S_3), the boundary condition is a perfect electric wall on the segment where the port is defined, and a perfect magnetic wall elsewhere.

The calculation of the entries of the (frequency–dependent) MoM matrix requires the initial determination of the (frequency–independent) coupling integrals between the basis functions and the electric modal vectors of the box

$$I_{rm}^{(p)} = \int_{S_p} \vec{e}_r^{(p)}(\vec{r}) \cdot \vec{\mathcal{E}}_m(\vec{r})\, dS \quad (3)$$

For sake of simplicity, in this work, we consider the patches attached to the box wall with a rectangular shape (like S_2 in Fig. 1): for them, $\vec{e}_r^{(p)}$ are expressed by analytical formulas ([3], Appendix II), and the coupling integrals (3) are known in closed form. On the contrary, the internal patches may have an arbitrary shape: in this case, the basis functions and the coupling integrals are calculated numerically, using the BI–RME method, as described in the next section.

III. BI–RME Calculation of Entire Domain Basis Functions and Coupling Integrals

As well known, the modal vectors $\vec{e}_r^{(p)}$ are subdivided in two classes, solenoidal ($\vec{e}_r^{\,\prime(p)}$) and irrotational ($\vec{e}_r^{\,\prime\prime(p)}$), which can be expressed in terms of scalar potentials

$$\vec{e}_r^{\,\prime(p)} = -\vec{u}_z \times \frac{\nabla_T \psi_r^{\prime(p)}}{\kappa_r^{\prime(p)}} \qquad \vec{e}_r^{\,\prime\prime(p)} = -\frac{\nabla_T \psi_r^{\prime\prime(p)}}{\kappa_r^{\prime\prime(p)}} \quad (4)$$

where $\psi_r^{\prime(p)}$ and $\psi_r^{\prime\prime(p)}$ are the solution of the homogeneous Helmholtz equation in the domain S_p with the Dirichelet and Neumann boundary condition, respectively, and $\kappa_r^{\prime(p)}$ and $\kappa_r^{\prime\prime(p)}$ are the corresponding eigenvalues.

The numerical solution of the homogeneous Helmholtz equation in the domain S_p can be efficiently accomplished by the BI–RME method, which provides $\{[\partial\psi_r^{\prime(p)}/\partial n_p]_{\partial S_p}, \kappa_r^{\prime(p)}\}$ and $\{[\psi_r^{\prime\prime(p)}]_{\partial S_p}, \kappa_r^{\prime\prime(p)}\}$ as eigensolutions of a linear matrix eigenvalue problem [4], [6], [7]. The potentials over the whole domain S_p can be obtained from these boundary values and, by using (4), the coupling integrals (3) can be directly calculated.

However, the application of the BI–RME method is even more convenient if the coupling integrals (3) are transformed from surface to line integrals. By using (4) and expressing $\vec{\mathcal{E}}_m$ through scalar potentials ($\chi_m^{\prime}$ for TM and $\chi_m^{\prime\prime}$ for TE modes, with the corresponding $k_m^{\prime}$ and $k_m^{\prime\prime}$ eigenvalues), and applying the second Green's identity, we obtained

$$\int_{S_p} \vec{e}_r^{\,\prime(p)} \cdot \vec{\mathcal{E}}_m^{\prime}\, dS = 0 \quad (5)$$

$$\int_{S_p} \vec{e}_r^{\,\prime\prime(p)} \cdot \vec{\mathcal{E}}_m^{\prime}\, dS = \frac{\kappa_r^{\prime\prime(p)}}{k_m^{\prime}(\kappa_r^{\prime\prime(p)2} - k_m^{\prime 2})} \int_{\partial S_p} \psi_r^{\prime\prime(p)} \frac{\partial \chi_m^{\prime}}{\partial n_p}\, d\ell \quad (6)$$

$$\int_{S_p} \vec{e}_r^{\,\prime(p)} \cdot \vec{\mathcal{E}}_m^{\prime\prime}\, dS = \frac{k_m^{\prime\prime}}{\kappa_r^{\prime(p)}(\kappa_r^{\prime(p)2} - k_m^{\prime\prime 2})} \int_{\partial S_p} \chi_m^{\prime\prime} \frac{\partial \psi_r^{\prime(p)}}{\partial n_p}\, d\ell \quad (7)$$

$$\int_{S_p} \vec{e}_r^{\,\prime\prime(p)} \cdot \vec{\mathcal{E}}_m^{\prime\prime}\, dS = \frac{1}{\kappa_r^{\prime\prime(p)} k_m^{\prime\prime}} \int_{\partial S_p} \psi_r^{\prime\prime(p)} \frac{\partial \chi_m^{\prime\prime}}{\partial t_p}\, d\ell \quad (8)$$

where $\vec{n}_p$ is the outward normal vector on ∂S_p, and $\vec{t}_p = \vec{u}_z \times \vec{n}_p$.

The evident advantage of this transformation comes from the possibility of calculating the coupling integrals by a one–dimensional numerical integration. Furthermore, the contour integrals involve $[\partial\psi_r^{\prime(p)}/\partial n_p]_{\partial S_p}$, $[\psi_r^{\prime\prime(p)}]_{\partial S_p}$, $\kappa_r^{\prime(p)}$, and $\kappa_r^{\prime\prime(p)}$, which are the output of the BI–RME method. Therefore, the use of the BI–RME method actually removes the need of knowing the current on the metallic surfaces (standard MoM with rooftops) and replaces it with the current distribution of the modes on the boundary of each metallic area. In other words, there is one more transformation introduced by the BI–RME method that moves the problem from the surfaces to the boundaries with a very substantial time saving.

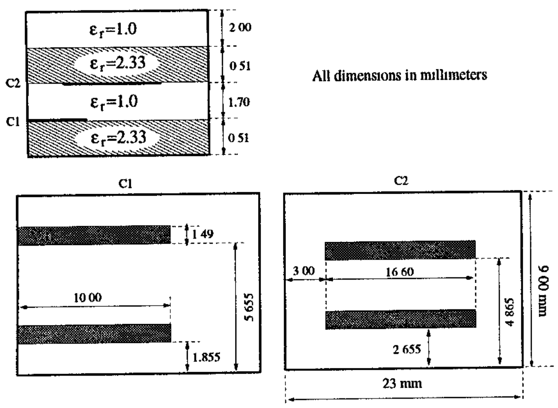

Fig. 2. Geometry of the broadside coupled filter used in validating the proposed technique.

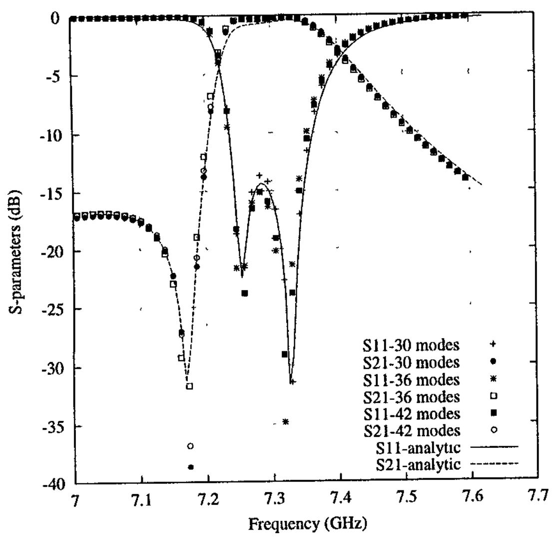

Fig. 3. Comparison between results obtained with analytical coupling integrals and with the numerical coupling integrals derived in this paper, for the multilayered broadside coupled filter shown in Fig. 2 (number of modes in the IE-kernel is fixed to 2000 modes).

IV. Numerical Results

For validation purposes of the approach derived in this contribution, we investigated a planar filter with a transmission zero implemented below the pass–band of the filter following the procedure described in [8]. This filter is composed of two rectangular resonators broadside coupled with the input and output lines (see Fig. 2).

The circuit has been analyzed with the software code developed in [3] and with the approach derived in the present contribution, obtaining the results presented in Fig. 3. In the same figure convergence results with respect the number of basis functions included in each metallic area are shown. It can be seen that the agree-

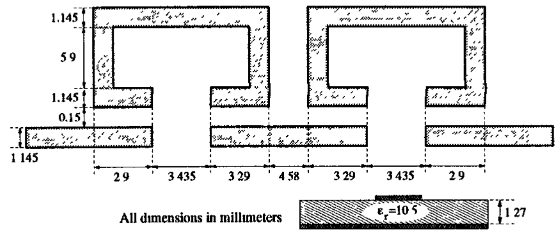

Fig. 4. Printed microstrip filter composed of two open–loop resonators.

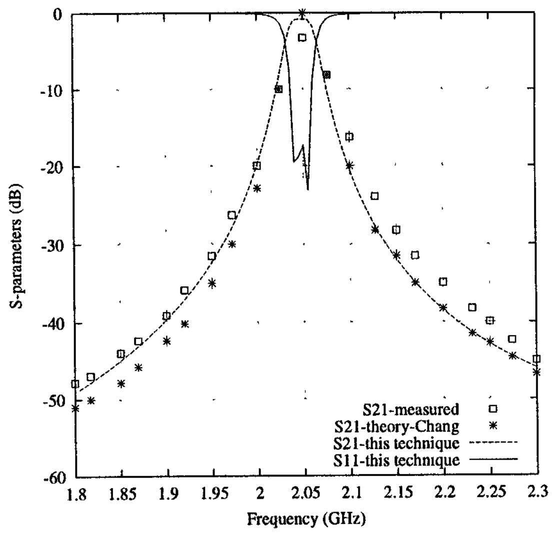

Fig. 5. Comparison between simulated and measured results presented in [9], and results obtained with the present approach, for the filter shown in Fig. 4.

ment between the two approaches is very good, therefore validating the numerical computation of the coupling integrals as proposed in this work. For this example, the analytical calculation of the coupling integrals (3) takes 2.5 *sec* (on a Pentium III at 550 MHz), while the numeric one takes 28 *sec*, when 30 modes are used in each metallic area and 2000 modes in the IE kernel. Since the frequency–by–frequency calculation of the filter response requires 0.76 *sec* per point, the total computing time for the analysis in 100 frequency points is 78.5 *sec* with analytical coupling integrals, and 104 *sec* with the numerical approach. Therefore, the numerical approach leads to a limited increase in the total simulation time required for the wideband analysis of the whole structure.

Once the numerical method has been validated, we used the code for the analysis of printed circuits involving resonators with complex shapes, which fully exploit the capabilities of the method. The second example refers to the analysis of a narrow–band filter composed of two open–loop resonators (Fig. 4), firstly proposed in [9]. In Fig. 5, measured results and simulations

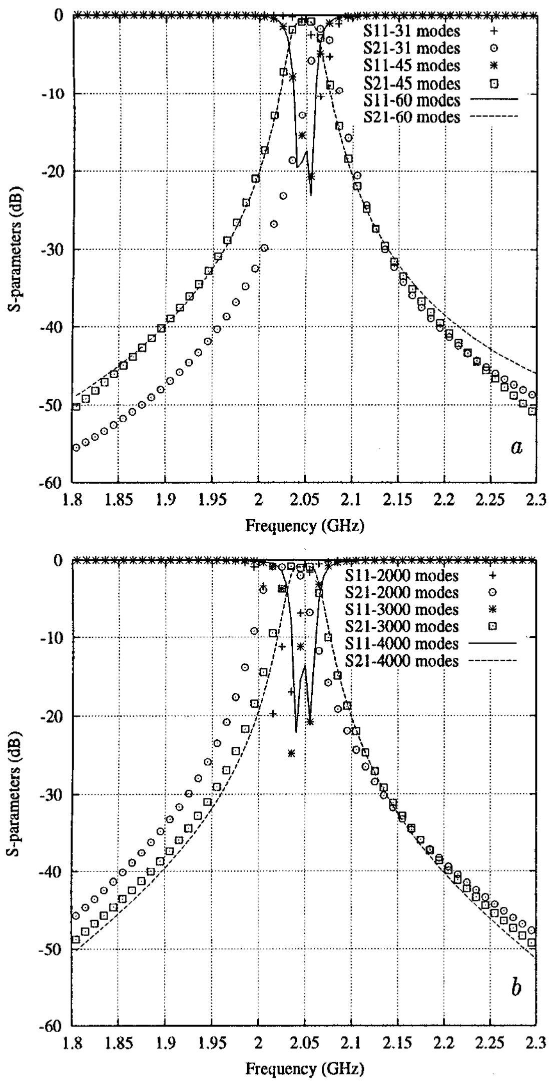

Fig. 6. Convergence behavior of the MoM/BI-RME method: *a*) as a function of the number of basis functions used in the open–loop resonators (number of modes in the IE kernel is fixed to 4000 modes); *b*) as a function of the number of modes used in the IE kernel (number of modes in the open-loop resonators is fixed to 45 modes).

given in [9] are compared with results obtained by the Mom/BI–RME approach showing good agreement. Finally, Fig. 6 shows the convergence properties of the numerical results as a function of the number of basis functions in each metallic area of the circuit, and as a function of the modes taken in the IE kernel. Results show that convergence is attained with about 45 basis functions in the open–loop resonators, and with about 3000 modes in the IE kernel. It is worthy observing that, when using 45 modes in the open–loop resonators and 3000 modes in the IE kernel, the code takes 120 *sec* for the initial calculation of the frequency independent coupling integrals, plus 1.5 seconds for each subsequent point in frequency (on a Pentium III at 550 MHz). Thus, the total computing time for the analysis in 100 frequency points is 270 *sec*.

V. Conclusion

In this contribution we have presented an efficient technique for the accurate analysis of multilayered shielded printed circuits composed of arbitrary shaped metallic areas. The technique is based on an Integral Equation solved by using the Method of Moments with entire domain basis functions. The basis functions are efficiently evaluated by the Boundary Integral–Resonant Mode Expansion method. This leads to MoM matrices of small size, even in the case of complex circuits. Moreover, the transformation of the coupling integrals from surface to line integrals, practically moves the problem unknown from the current distribution on the surface to its value on the boundary.

Results show that the approach is indeed feasible and leads to software codes which are both efficient an accurate. Comparisons with numerical and measured results have demonstrated the validity of the approach for the analysis of complex printed circuits, and good agreements have been obtained in all the cases treated.

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